Gradient

→ Vector quantity

→ Applied on scalar.

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z.$$

$$\nabla F = \frac{\partial F}{\partial x} a_x + \frac{\partial F}{\partial y} a_y + \frac{\partial F}{\partial z} a_z.$$

Q) Find gradient of the function $F = xy^2 + z^3 y$. at point $(1, 2, 3)$

$$\nabla F = \frac{\partial}{\partial x}(xy^2 + z^3 y) a_x + \frac{\partial}{\partial y}(xy^2 + z^3 y) a_y + \frac{\partial}{\partial y}(xy^2 + z^3 y) a_z.$$

$$\nabla F = \cancel{2xy}\, y^2 a_x + (2xy + z^3) a_y + 3z^2 y a_z.$$

at $(1, 2, 3)$

$$\nabla F = 4a_x + 31 a_y + 54 a_z.$$

Divergence.

→ Divergence is scalar.

→ applied on vector.

Function $F = f_1 a_x + f_2 a_y + f_3 a_z$

$$\nabla \cdot F = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}.$$

$a_x \cdot a_x = 1$ $\quad$ $a_x a_y = a_y a_z = a_z a_x = 0$

$a_y \cdot a_y = 1$

$a_z \cdot a_z = 1$

Q) Find the divergence of a function $F = xy^2 a_x + y a_y + xz a_z$ at point $(1,2,1)$.

$$\frac{\partial f_1}{\partial x} = y^2 \qquad \frac{\partial f_2}{\partial y} = 1 \qquad \frac{\partial f_3}{\partial z} = x$$

$$\nabla \cdot F = y^2 + 1 + x$$

$$= 4 + 1 + 1 = 6$$

Note :- To find out whether F is solenoid

$$\boxed{\text{Proof :- } \nabla \cdot H = 0}$$

CURL :-

→ Curl is a vector

→ applied on vector.

$$F = f_1 a_x + f_2 a_y + f_3 a_z$$

$$\nabla \times F = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

Properties :- $a_x \times a_x = a_y \times a_y = a_z \times a_z = 0$

$$a_x \times a_y = a_z$$

$$a_y \times a_z = a_x$$

$$a_z \times a_x = a_y$$

If not in order answer will have a -ve sign.

Q) Show that the vector $\cancel{H = 3y^4 z^2 a_x + 4x^3 z}$

$H = yz a_x + xz a_y + xy a_z$ is both solenoidal and ~~irrotational~~ irrotational

Note :- If irrotational

$$\nabla \times H = 0$$

$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix}$$

$$= a_x \left[\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(xz)\right] - a_y \left[\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial z}(yz)\right] + a_z \left[\frac{\partial}{\partial x}(xz) - \frac{\partial}{\partial y}(yz)\right].$$

$$= a_x [x - x] - a_y [y - y] + a_z [z - z]$$

$$= \underline{\underline{0}}$$

$\Rightarrow \nabla \times H = 0$.

i.e, the function is irrotational.

THREE TYPES OF COORDINATE SYSTEM.

> Cartesian $[x, y, z]$
> Cylinderical $[\rho, \phi, z]$
> Spherical $[r, \theta, \phi]$

Relationship b/w. cartesian coordinate system and cylindrical coordinate system.

$\rho = \sqrt{x^2 + y^2}$ $\quad \phi = \tan^{-1}(y/x)$ $\quad z = z$.

$x = \rho \cos\phi$ $\quad y = \rho \sin\phi$ $\quad z = z$.

Relationship b/w (A_x, A_y, A_z) and (A_ρ, A_ϕ, A_z)

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$a_\rho = \cos\phi\, a_x + \sin\phi\, a_y$
$a_\phi = -\sin\phi\, a_x + \cos\phi\, a_y$
$a_z = a_z$.

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$a_x = \cos\phi\, a_\rho - \sin\phi\, a_\phi$
$a_y = \sin\phi\, a_\rho + \cos\phi\, a_\phi$
$a_z = a_z$.

Relationship b/w cartesian coordinate system and spherical coordinate.

$r = \sqrt{x^2+y^2+z^2}$ $\quad \theta = \tan^{-1}\dfrac{\sqrt{x^2+y^2}}{z}$ $\quad \phi = \tan^{-1}\dfrac{y}{x}$

$x = r\sin\theta\cos\phi$ $\quad y = r\sin\theta\sin\phi$ $\quad z = r\cos\theta$.

Relationship b/w (A_x, A_y, A_z) and (A_r, A_θ, A_ϕ)

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

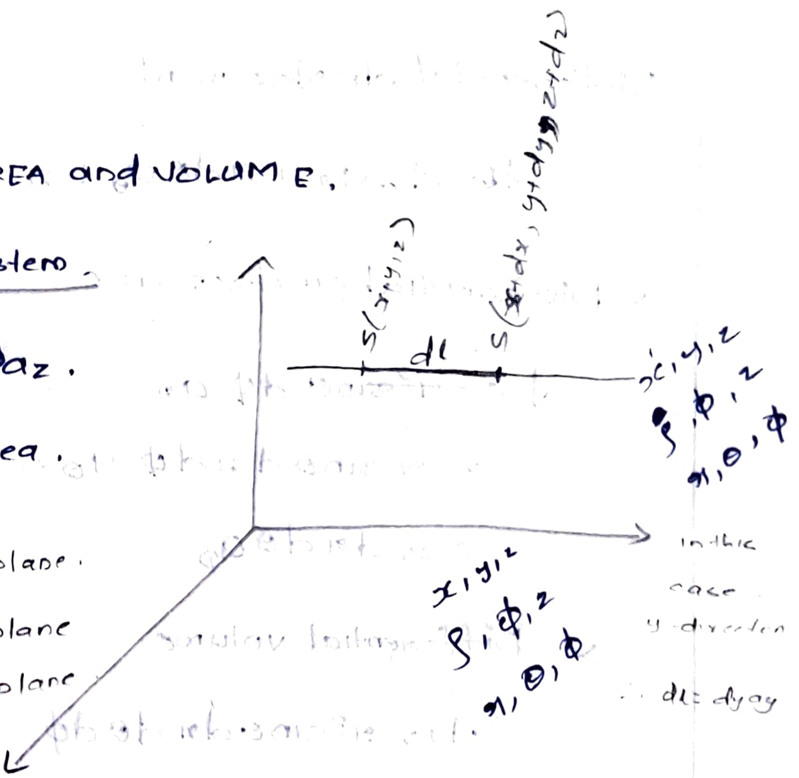
VECTOR CALCULUS.

DIFFERENTIAL, LENGTH, AREA and VOLUME.

Cartesian coordinate system.

Differential displacement

$dl = dx\,a_x + dy\,a_y + dz\,a_z$.



Differential surface Area.

$$ds = dy\,dz\,a_x \} - yz \text{ plane}$$
$$= dx\,dz\,a_y \} xz \text{ plane}$$
$$= dx\,dy\,a_z \} xy \text{ plane}$$

Differential volume

$$dv = dx\,dy\,dz.$$

Cylindrical coordinate system :-

Differential displacement.

$$dl = d\rho\,a_\rho + \rho\,d\phi\,a_\phi + dz\,a_z.$$

Differential surface area.

$$ds = \rho\,d\phi\,dz\,a_\rho \rightarrow \rho \text{ constant}$$
$$= d\rho\,dz\,a_\phi - \phi \text{ constant}$$
$$= \rho\,d\rho\,d\phi\,a_z. - z \text{ constant}$$

Differential volume

$$dv = \rho\,d\rho\,d\phi\,dz.$$

Spherical coordinate system,

Differential displacement

$$dl = dr\, a_r + r\, d\theta\, a_\theta + r\sin\theta\, d\phi\, a_\phi$$

> Differential surface are

$$dS = r^2 \sin\theta\, d\phi\, a_r \quad - r \text{ constant}$$

$$= r\sin\theta\, dr\, d\phi\, a_\theta \cdot \quad \theta \text{ constant}$$

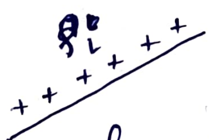
$$= r\, dr\, d\theta\, a_\phi \quad - \phi \text{ constant}$$

> Differential volume

$$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

LINE CHARGE DENSITY (ρ_L)

surface integral also known as net outward flux of A from S.



$$\rho_L = \frac{dQ}{dL}.$$

$$dQ = \rho_L\, dl.$$

$$Q = \int_L \rho_L\, dl..$$

SURFACE CHARGE DENSITY (ρ_s)

$$\rho_s = \frac{dQ}{ds}$$

$$dQ = \rho_s\, ds.$$

$$Q = \int_S \rho_s\, ds.$$

Volume charge density (ρ_v)

$$\rho_v = \frac{dQ}{dv}.$$

$$dQ = \rho_v \, dv$$

$$Q = \int_v \rho_v \, dv$$

Divergence theorem.

The divergence theorem states that the total outward flux of a vector field A through the closed surface S is the same as the volume integral of the divergence of A.

$$\oint_S A \cdot ds = \int_v \nabla \cdot A \, dV$$

$$\oint_L A \, dl = \int_s \nabla \times A \, ds.$$

Stoke's theorem

$$\oint_L A \cdot dl = \int_S (\nabla \times A) \, ds.$$

Stokes theorem states that the circulation of a vector field A around a closed path L is equal to the surface integral of the curl of A over the open surface S bounded by L provided A and $\nabla \times A$ are continuous on S.

"Electric field intensity or electric field strength (E) is the force by unit charge when placed in an Electric field."

$$E = \lim_{Q \to 0} \frac{F}{Q} \quad \text{or} \quad \boxed{E = \frac{F}{Q}}$$

Electric flux density (D)

$$D = \varepsilon_0 E.$$

The flux due to the electric field E can be calculated by using the general flux equation.

$$\psi = \int_S A \cdot ds.$$

Electric field intensity is depending on the medium in which the charge is placed

The vector field D is defined as.

$$D = \varepsilon_0 E$$

and the electric flux ψ in terms of D

$$\psi = \int_S D \cdot ds.$$

Gauss's Law.

Gauss's law states that the total electric flux ψ through any closed surface is equal to the total charge enclosed by the surface

$$\psi = Q_{enc}.$$

$$\psi = \oint_S d\psi = \oint_S D \cdot ds.$$

$Q_{enc} = \int_V \rho_v \, dv$

$$\oint_S D \cdot ds = \int_V \rho_v \, dv.$$

ρ_v = volume charge density

applying divergence theorem

$$\int_V \nabla \cdot D \, dv = \int_V \rho_v \, dv$$

$$\nabla \cdot D = \rho_v$$

First maxwell's equation

Maxwell's equation states that the volume charge density is the same as the divergence of electric flux density.

AMPERE'S CIRCUIT LAW

Ampere's circuit law states that the line integral of the tangential components of H around a closed path is the same as the net current I_{enc} by the path.

$$\oint_L H \cdot dl = I_{enc}$$

$$I_{enc} = \int_S J \cdot ds$$

where J is the volume current density

Applying Stoke's theorem in LHS

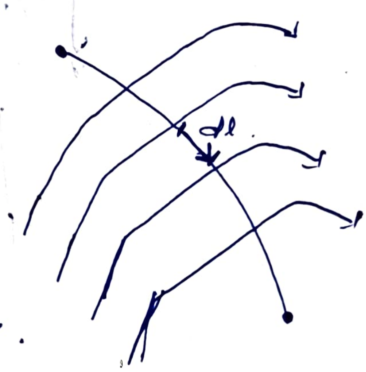
$$\oint H \cdot dl = \int_S (\nabla \times H)\, ds$$

$$\int_S (\nabla \times H)\, ds = \int_S J\, ds$$

$$\boxed{\nabla \times H = J}$$ Third Maxwell's equation

POTENTIAL DIFFERENCE [V]

From coulomb's law, the force on Q is $F = QE$ so the work done in displacing the charge by dl is



The total work done

$$W = -Q\int_A^B E \cdot dl$$

Dividing W by Q gives the potential energy per unit charge.

$$V_{AB} = \frac{W}{Q} = -\int_A^B E \cdot dl$$

MODULE - 2.

Current :- Rate of movement of charge passing through a reference point.

$$I = \frac{dQ}{dt}.$$

Total current crossing the total surface is given as

$$I = \int_S J \cdot ds.$$

Principle of charge conservation.

The outward flow of charge must be balanced by a rate of decrease of charge within the closed surface.

$$I_{out} = \oint_l J \cdot ds = \frac{-dQ}{dt}.$$

Q is the charge enclosed by the circle.

Applying divergence theorem.

$$\oint_S J \cdot ds = \int_V \nabla \cdot J dv.$$

$$\frac{-dQ_{in}}{dt} = \frac{-d}{dt} \int_V \rho_v dv = -\int \frac{\partial \rho_v}{\partial t} dv$$

$$\int_V \nabla \cdot J \cdot dv = -\int \frac{\partial \rho_v}{\partial t} dv$$

$$\boxed{\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}}$$

continuity equation.

DISPLACEMENT CURRENT DENSITY.

From Ampere's circuit law for static electromagnetic field.

$$\nabla \times H = J \quad \ldots (1)$$

Third maxwell's equation.

Divergence of a curl of any vector field is zero.

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J = 0 \quad \text{——} (2)$$

But from continuity equation.

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \neq \cancel{0} \quad \text{——} (3)$$

To satisfy the above condition, we modify the equation.

$$\nabla \times H = J + J_d \quad \text{——} (4)$$

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J + \nabla \cdot J_d = 0.$$

$$\nabla \cdot J_d = -\nabla \cdot J$$

$$\nabla \cdot J_d = -\nabla \cdot J$$

$$\nabla \cdot J_d = \frac{\partial \rho_v}{\partial t}.$$

$$= \frac{\partial}{\partial t}(\nabla \cdot D)$$

$$\nabla \cdot J_d = \nabla \cdot \frac{\partial D}{\partial t}$$

$$J_d = \frac{\partial D}{\partial t}$$ Displacement current density.

$$\boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$$

Which is the Maxwell's equation.

for a time varying feild, where J is the conduction current

$$J = \sigma E$$

Based on displacement current density we define displacement current as.

$$I_d = \int J_d \cdot ds = \int \frac{\partial D}{\partial t} \cdot ds.$$

MAGNETIC FLUX DENSITY

→ The magnetic flux density B is similar to the electric flux density D.

$B = \mu_0 H$ $\mu_0 \rightarrow$ permiability of free space $\left(\frac{\text{Henrys}}{\text{meter}}\right)$

$\mu_0 = 4\pi \times 10^{-7} H/m$.

→ Magnetic flux through Surface S is given by

$$\psi = \int_S B \cdot ds$$ Unit → Webers (Wb)

Magnetic flux density → Wb/m^2.

An isolated magnetic charge does not exist.

$$\oint B \cdot ds = 0.$$

$$\oint_S A \cdot ds = \int_V \nabla \cdot A \, dv$$

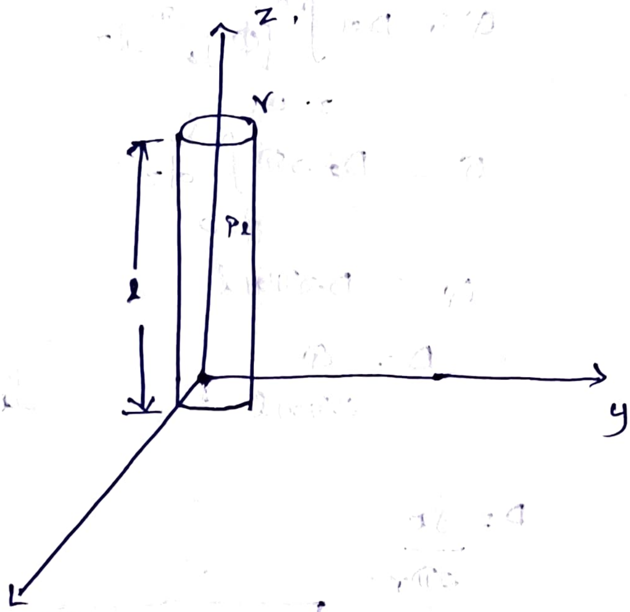
$$\oint B \cdot ds = \int_V \nabla \cdot B \, dv = 0$$

$$\nabla \cdot B = 0$$

Ap

APPLICATION OF GAUSS LAW

Electric field intensity due to infinite line charge



Consider an infinite line charge of ρ_l c/m as the z axis of cylindrical coordinate.

- Consider a circular cylinder of radius r with length 'l' as Gaussian surface as shown in figure.
- The electric flux density D is normal to the surface of the cylinder at everywhere.
- By applying Gauss law to the closed surface of the cylinder

$$Q = \oint D \cdot ds$$

$$= \int_{top} D \cdot ds + \int_{bottom} D \cdot ds + \int_{sides} D \cdot ds$$

$$= 0 + 0 + \int_{sides} D \cdot ds \qquad \int_{top} D \cdot ds = \int_{bottom} D \cdot ds = 0.$$

$$= \int_{sides} D \cdot ds$$

$$Q = D\int ds = D\int_{z=0}^{l}\int_{\phi=0}^{2\pi} r\, d\phi\, dz.$$

$$Q = Dr\int_{z=0}^{l} [\phi]_0^{2\pi}\, dz.$$

$$Q = Dr2\pi\int_{z=0}^{l} dz$$

$$Q = D2\pi r l$$

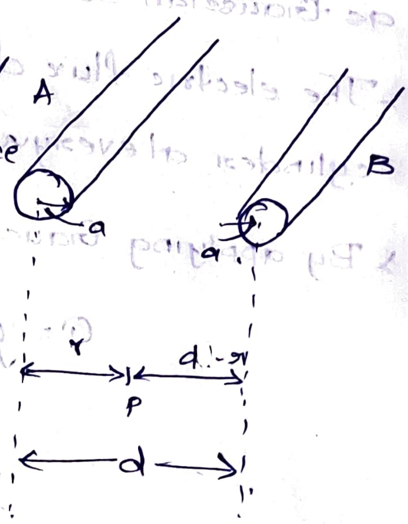
$$D = \frac{Q}{2\pi r l} \qquad \rho_l = \frac{Q}{l}.$$

$$D = \frac{\rho_l}{2\pi r}.$$

$$D = \varepsilon_0 E \qquad \boxed{E = \frac{D}{\varepsilon_0} = \frac{\rho_l}{2\pi\varepsilon_0 r}}$$

(7) Capacitance of 2 wire transmission line

Consider a two parallel conductor of radius, 'a' seperated by a distance 'd' as shown in figure.



> If the conductor. A has charge of ρ_l c/m along its length, this will induce a charge of $-\rho_l$ c/m on the conductor B.

> The electric field intensity at any point P with a distance r from the conductor 'A' is the algebraic sum of electric field intensity at P due to conductor A and conductor B.

$$E = \frac{\rho_l}{2\pi\varepsilon x} - \frac{\rho_l}{2\pi\varepsilon(d-x)}$$

$$= \frac{\rho_l}{2\pi\varepsilon}\left[\frac{1}{x} - \frac{1}{d-x}\right]$$

The potential difference b/w the conductors is given by

$$V = -\int E \cdot dr.$$

$$= -\int_{d-a}^{a} \frac{\rho_l}{2\pi\varepsilon}\left[\frac{1}{x} - \frac{1}{d-x}\right] dx.$$

$$= \cancel{-\int_{d-a}^{a}} \frac{-\rho_l}{2\pi\varepsilon}\int_{d-a}^{a}\left[\frac{1}{r} - \frac{1}{d-x}\right] dx.$$

$$= \frac{-\rho_l}{2\pi\varepsilon}\left[\ln[x]_{d-a}^{a} - \ln.[d-x]_{d-a}^{a}\right]$$

$$= \frac{-\rho_l}{2\pi\varepsilon}\left[\ln[a-(d-a)] - \ln\left[d-a-(d-(d-a))\right]\right]$$

$$= \frac{-\rho_l}{2\pi\varepsilon}\left[\ln(a) - \ln(d-a)\right] - \left[\ln(d-a) - \ln(a)\right].$$

$$= \frac{-\rho_l}{2\pi\varepsilon}\left[\ln\left(\frac{a}{d-a}\right) - \ln(d-a) + \ln(a)\right]$$

$$= \frac{-\rho_l}{\cancel{2}\pi\varepsilon}\left[\cancel{2}\ln\left(\frac{a}{d-a}\right)\right] = \frac{-\rho_l}{\pi\varepsilon}\left[-\ln\left(\frac{d-a}{a}\right)\right]$$

$$= \frac{\rho_l}{\pi\varepsilon}\left[\ln\left(\frac{d-a}{a}\right)\right]$$

Capacitance per unit length.

$$C = \frac{\rho_L}{V}$$

$$= \frac{\rho_L}{\frac{\rho_L}{\pi\varepsilon}\left[\ln\left(\frac{d-a}{a}\right)\right]}$$

$$= \frac{\pi\varepsilon}{\ln\left(\frac{d-a}{a}\right)} \text{ F/m}$$

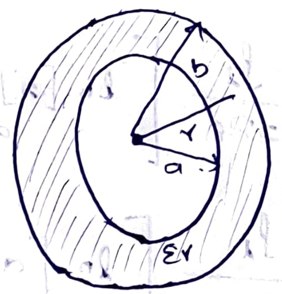
$$C = \frac{Q}{V} = \frac{Q/L}{V} = \frac{\rho_L}{V}$$

If $d >> a$.

$$\boxed{C = \frac{\pi\varepsilon}{\ln(d/a)}} \text{ F/m.}$$

CAPACITANCE OF COAXIAL CABLE.

Consider a coaxcial cable of inner radius 'a' and outer radius 'b' as shown in figure.



> The relative permitivity of di-electric filled in b/w two coaxcial cylinders is ε_r.

> A potiential diff^n V is applied in between two cylinders.

> The two cylinders are charged at the rate of ρ_L c/m.

> It is assumed that the inner cylinder has charge of ρ_L c/m and outer cylinder has a charge of $-\rho_L$ c/m.

> By applying Gauss law, the electric field E at any distance r from the axis of cylinder is given by $E = \frac{\rho_L}{2\pi\varepsilon r}$

> The potential diff b/w two coaxial cable is,

$$V = -\int_b^a E \cdot dr.$$

$$= -\int_b^a \frac{\rho_l}{2\pi\varepsilon r}\,dr = \frac{-\rho_l}{2\pi\varepsilon}\int_b^a \frac{dr}{r}.$$

$$= -\frac{\rho_l}{2\pi\varepsilon r}\ln[r]_b^a = -\frac{\rho_l}{2\pi\varepsilon r}\ln\left[\frac{a}{b}\right]$$

$$\boxed{V = \frac{\rho_l}{2\pi\varepsilon r}\ln\left[\frac{b}{a}\right]}$$

$$C = \frac{Q}{V} = \frac{Q/l}{V} = \frac{\rho_l}{V}$$

Capacitance of coaxial cable per unit length is

$$C = \frac{\rho_l}{V} = \frac{\rho_l}{\frac{\rho_l}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right)}$$

$$\boxed{C = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)}} \; F/m \qquad \boxed{\varepsilon = \varepsilon_0\varepsilon_r}$$

MAGNETIC POTENTIAL

Magnetic potential.

- Scalar Magnetic Potential (V_m) ($J = 0$)
- Vector Magnetic Potential (A) ($J \neq 0$)

> Scalar Magnetic potential is defined only in a region where $J=0$ ~~i.e, no~~ that is no current source is present.

> It is the quantity whose negative gradient gives the magnetic intensity.

$$-\nabla V_M = H.$$

$$-V_M = \int H \cdot dl$$

$$V_m = -\int H \cdot dl.$$

$$H = -\nabla V_M.$$

Taking curl on both sides.

$$\nabla \times H = -\nabla \times (\nabla V_M)$$

$$\Rightarrow \nabla \times H = 0. \quad \text{--- (1)}.$$

Identity

$$\nabla \times (\nabla A) = 0$$

$$\nabla \cdot (\nabla \times A) = 0.$$

From Ampere's Law

$$\nabla \times H = J \quad \text{--- (2)}.$$

Comparing equation (1) & (2).

$$J = 0.$$

Scalar potential satisfies Laplace equation

$$\nabla^2 V_M = 0.$$

From Maxwell's equation, we know $\nabla \cdot B = 0$.

for free space $B = \mu_0 H$.

$$\nabla (\mu_0 H) = 0.$$

$$\mu_0 \nabla \cdot (H) = 0.$$

Now, $H = -\nabla V_m.$

$$\mu_0 \nabla \cdot (-\nabla V_M) = 0$$

$$-\nabla(\nabla \cdot V_M) = 0.$$

$$\nabla^2 V_M = 0.$$

WAVE EQUATION

Application of Maxwell equation gives wave equation,

Assume magnetic and electric field present in a medium varying with time.

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad \text{—— (1)}$$

$$J = \sigma E.$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad \text{—— (2)}$$

$$B = \mu H.$$

$$\nabla \times H = \sigma E + \frac{\partial D}{\partial t} \quad \text{—— (3)}$$

$$(\nabla \cdot D) V$$

$$\nabla \cdot DV - D \cdot \nabla V$$

$$\nabla \times E = -\mu \frac{\partial}{\partial t}(H). \quad \text{—— (4)}$$

(4) curl apply.

$$\nabla \times (\nabla \times E) = -\mu \frac{\partial}{\partial t}(\nabla \times H) \quad \text{—— (5)}$$

(3) in (5)

$$\nabla \times (\nabla \times E) = -\mu \frac{\partial}{\partial t}\left(\sigma E + \frac{\partial D}{\partial t}\right),$$

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A.$$

$$\nabla \cdot (\nabla \cdot E) - \nabla^2 E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \frac{\partial D}{\partial t}\right)$$

At charge free medium $\nabla \cdot E = 0.$

$$-\nabla^2 E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \frac{\varepsilon \partial E}{\partial t}\right).$$

$$\boxed{\begin{aligned}\nabla^2 E &= \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \\ \nabla^2 H &= \mu\sigma \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}\end{aligned}}$$

- At free space $\sigma = 0$.

$$\boxed{\begin{aligned}\nabla^2 E &= \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \\ \nabla^2 H &= \mu\varepsilon \frac{\partial^2 H}{\partial t^2}\end{aligned}}$$

Phasor form

$$\nabla^2 E_s = j\omega\mu(\sigma + j\omega\varepsilon) E_s.$$

$$\nabla^2 H_s = j\omega\mu(\sigma + j\omega\varepsilon) H_s.$$

MODULE-3

Loceless,

$$E = E_0 \cos(\omega t - \beta z)\, a_x$$

phase constant

a_x → direction of electric field.

z → direction of wave propogation.

E_0 ↑ amplitude.

ω ↑ frequency.

$$H = H_0 \cos(\omega t - \beta z)\, a_y$$

→ -ve sign indicates wave propogation in +ve z direction.

Q) An EM wave is designated by $E = 30\cos(2\pi \times 10^6 t - \beta x)\cdot a_y$ is passing through a losseless dielectric medium of $\mu_r = 3$, $\varepsilon_r = 27$ having frequency 200MHz. Determine β, λ, V_P, η.

Ans).

Medium.

$$E = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_x \quad -\theta_n$$

$$H = H_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_y \quad -\theta_n$$

Wave propogation in Lossy Dielectric.

Propogation const $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

Attenuation const $\alpha = \omega\sqrt{\dfrac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\dfrac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$

Phase constant $\beta = \omega\sqrt{\dfrac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\dfrac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$

Intrinsic Impedance.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

Wavelength,

$$\lambda = \frac{2\pi}{\beta}$$

Velocity, $v = \frac{\omega}{\beta}$.

Plane wave in Lossless dielectric

In lossless dielectric $\sigma << \omega\varepsilon$.

$\sigma \simeq 0$, $\varepsilon = \varepsilon_0 \varepsilon_r$, $\mu = \mu_0 \mu_r$.

$\gamma = j\omega\sqrt{\mu\varepsilon}$. $\qquad \gamma = \alpha + j\beta$.

$\alpha = 0$

$\beta = \omega\sqrt{\mu\varepsilon}$.

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}} = \frac{2\pi}{2\pi f\sqrt{\mu\varepsilon}} = \frac{1}{f\sqrt{\mu\varepsilon}}.$$

$$v = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}}.$$

Plane wave in free space:

$\sigma = 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$.

$\alpha = 0$

$\Rightarrow \beta = \omega\sqrt{\mu_0\varepsilon_0}$, $\quad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$.

$\therefore \beta = \frac{\omega}{c}$.

$$\boxed{\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \text{ it can be written as. } \eta = 120\pi \text{ or } 370\Omega.}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$$

$$= 120\pi \sqrt{\frac{\mu_r}{\varepsilon_r}} \Big/ 377\Omega \sqrt{\frac{\mu_r}{\varepsilon_r}}.$$

Wave equation

$$\nabla^2 H = \mu\varepsilon \frac{\partial^2 H}{\partial t^2}.$$

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}.$$

$$\frac{1}{c^2} = \mu_0 \varepsilon_0.$$

$$\nabla^2 H = \frac{1}{c^2}\frac{\partial^2 H}{\partial t^2}.$$

$$\lambda = \frac{2\pi}{\beta}.$$

$$v = \frac{\omega}{\beta} = \frac{\omega}{\frac{\omega}{c}} = c.$$

Plane Waves in Good conductor.

$\sigma \simeq \infty$ $\quad \varepsilon = \varepsilon_0, \; \mu = \mu_0 \mu_r.$

$\sigma \gg \omega\varepsilon.$ $\quad$ there $\frac{\sigma}{\omega\varepsilon} \gg 1$

$$\alpha = \beta = \sqrt{\frac{\sigma\mu\omega}{2}}.$$

$$\eta = \sqrt{\frac{j\omega\varepsilon}{\sigma}}.$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ.$$

$$\lambda = \frac{2\pi}{\beta}$$

$$v = \frac{\omega}{\beta}.$$

Q) An EM wave is designated by $E = 30\cos(2\pi\times10^{6}t - \beta x)\, a_y$ is passing through a lossless dielectric medium of $\mu_r = 3$ & $\varepsilon_r = 27$ having frequency $f = 200\,MHz$, Determine. β, λ, V_p & η.

$E_0 = 30$, $\omega = 2\pi\times10^{6}$.

$\beta = \omega\sqrt{\mu\varepsilon}$. $\quad \mu = \mu_0\mu_r \quad \varepsilon = \varepsilon_0\varepsilon_r$.

$\mu_0 = 4\pi\times10^{-7}$. $\quad \varepsilon_0 = 8.854\times10^{-12}$.

$$\frac{1}{\sqrt{\mu_0\varepsilon_0}} = c = 3\times10^{8}\,m/s.$$

$$\beta = \omega\times\sqrt{\mu_0\varepsilon_0}\,\sqrt{\mu_r\varepsilon_r}$$

$$\beta = \frac{2\pi\times10^{8}}{3\times10^{8}}\times\sqrt{3\times27} = \cancel{18.84}\ 188.4\ rad/m.$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\cancel{18.84}\ 188.4} = 33.3m.$$

$$V_p = \frac{\omega}{\beta} = 333502.4\,m/s.$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\times\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$= 120\pi\times\sqrt{\frac{3}{27}} = 125.66.$$

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\ \cancel{\sqrt{\frac{\mu_r}{\varepsilon_r}}}$$

Q) A uniform plane wave propogating in a medium ~~$E = 2e^{-2z}\sin$~~

$E = 2e^{-2z}\sin(10^{8}t - \beta z)\, a_y\ V/m$. If the medium is characterized by ~~ε_r~~ $\varepsilon_r = 1$ and $\mu_r = 20$ & $\sigma = 3\,s/m$.

Find α, β & H.

$\omega = 10^{8}$

$\varepsilon = \varepsilon_0\varepsilon_r = 8.854\times10^{-12}$.

$$\frac{\sigma}{\omega\varepsilon} = \frac{3}{10^{8}\times8.854\times10^{-12}}$$

$$\alpha = \sqrt{\frac{\sigma \mu \omega}{2}} = \beta = 61.4.$$

$$H = H_0 e^{-\alpha z} \cos(\omega t - \beta z - \theta_n) a_y.$$

$\eta = \frac{E_0}{H_0}$.

$$\eta = \sqrt{\frac{\omega \mu}{\sigma}} = \sqrt{\frac{4\pi \times 10^{-7} \times 20 \times 10^{8}}{3}}$$

$H_0 = \frac{E_0}{\eta}$.

$$= 28.9440.$$

$$E_0 = 2.$$

$$H_0 = \frac{2}{28.9440}$$

$$= 69.1 \times 10^{-3}$$

$$\tan 2\theta_n = \frac{\sigma}{\omega \varepsilon} \rightarrow \text{loss tangent}.$$

~~$2\theta_n = t$~~ $\tan 2\theta_n = 3390.$

$$2\theta_n = \tan^{-1}(3390).$$

$$\theta_n = \frac{1}{2} \tan^{-1}(3390).$$

$$= 45°$$

$a_z \times a_y = -a_x.$

$= -61.42.$

$$H = -69.1 \times 10^{-3} e \sin(10^8 t + 61.42 - 45)\, a_x$$

$x\ y\ z$

$a_x \times a_y = a_z$

$a_y \times a_z = a_x.$

$a_z \times a_x = a_y.$

$a_E\ a_H\ a_K.$

$a_E \times a_H = a_K$

$a_H \times a_K = a_E$

$a_K \times a_E = a_H$

Reflection of a plane wave at oblique Incidence.

~~A wave travelling in a~~

> Electric field of a wave travelling in an arbitrary direction.

$$E = E_0 \cos(k r - \omega t)$$

k [propogation vector/ wave number vector]

$$= k_x a_x + k_y a_y + k_z a_z .$$

$r = x a_x + y a_y + z a_z \rightarrow$ position vector.

$k r = k_x x + k_y y + k_z z \rightarrow$ const.

If we consider a lossless medium, & EM wave is propogating in z-direction.

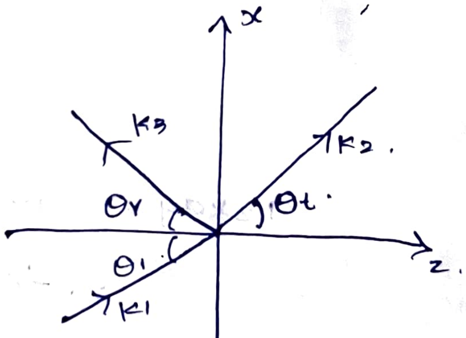
$$k_z = \beta .$$

$$k^2 = k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \varepsilon = \beta^2 \text{ (lossless medium)}.$$

Find the components

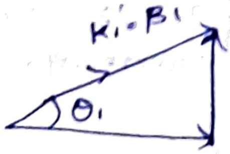
k_i, k_r & k_t w.r.t to x

x, y and z axis.



Medium 1 $(\varepsilon_1, \mu_1, \beta_1)$

Medium 2 $(\varepsilon_2, \mu_2, \beta_2)$

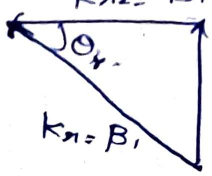


$k_{ix} = k_i \sin\theta_i$

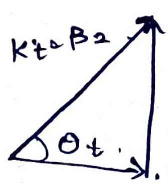
$= \beta_1 \sin\theta_i$

$k_{iz} = k_i \cos\theta_i = \beta_1 \cos\theta_i$

$k_{rz} = \beta_1 \cos\theta_r .$



$k_{rx} = \beta_1 \sin\theta_r .$



$k_{tx} = \beta_2 \sin\theta_t .$

$k_{tz} = \beta_2 \cos\theta_t .$

Incident wave

$$E_i = E_{i0} \cos(k_{ix} x + k_{iy} y + k_{iz} z - \omega_i t)$$

Reflected wave.

$$E_r = E_{r0} \cos(k_{rx} x + k_{ry} y + k_{rz} z - \omega_r t)$$

Transmitted wave

$$E_t = E_{t0} \cos(k_{tx} x + k_{ty} y + k_{tz} z - \omega_t t).$$

Since the tangential component of E must be continuous across the boundary $z = 0$.

$$E_i(z=0) + E_r(z=0) = E_t(z=0).$$

This boundary condition can be satisfied by the wave, for all x & y only if

1) $\omega_i = \omega_r = \omega_t = \omega$.
2) $k_{ix} = k_{rx} = k_{tx} = k_x$.
3) $k_{iy} = k_{ry} = k_{ty} = k_y$

$k_{ix} = k_{rx}$.

$k_i \sin\theta_i = k_r \sin\theta_r$.

k_i & k_r is in medium 1.

$k_i = k_r = \beta_1$

$\beta_1 \sin\theta_i = \beta_1 \sin\theta_r$

$\sin\theta_i = \sin\theta_r$.

$\theta_i = \theta_r$.

$k_{ix} = k_{tx}$

$k_i \sin\theta_i = k_t \sin\theta_t$

$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$.

$\beta_1 = \omega\sqrt{\mu_1\varepsilon_1}$ $\quad \beta_2 = \omega\sqrt{\mu_2\varepsilon_2}$.

$\omega\sqrt{\mu_1\varepsilon_1}\sin\theta_i = \omega\sqrt{\mu_2\varepsilon_2}\sin\theta_t$.

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\sqrt{\mu_1\varepsilon_1}}{\sqrt{\mu_2\varepsilon_2}}.$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{c\sqrt{\mu_1\varepsilon_1}}{c\sqrt{\mu_2\varepsilon_2}}.$$

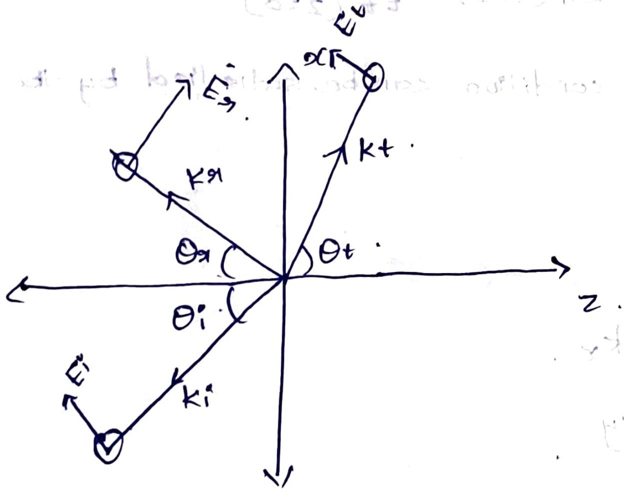
$$c\sqrt{\mu_1\varepsilon_1} = n_1 \quad c\sqrt{\mu_2\varepsilon_2} = n_2.$$

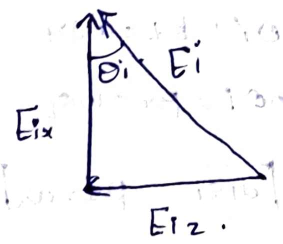
$n_1 > n_2$ - refractive indexes of medium 1 and 2 respectively.

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2}.$$

$$\boxed{n_2 \sin\theta_t = n_1 \sin\theta_i}$$

Oblique Incidence - Parallel Polarization.





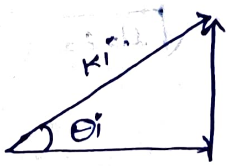
$E_{isx} = E_{io}\cos\theta_i a_x$.

$E_{isz} = -E_{io}\sin\theta a_z$.

$E_{is} = E_{io}\left[\cos\theta_i a_x - \sin\theta_i a_z\right]e^{-jkr}$.

~~$E_{is} = E_{io}[\cos\theta_i a_x - \sin\theta_i a_z]e^{-jkr}$~~

~~let~~



$k_{ix} = \beta_1 \sin\theta_i a_x$.

$k_{iz} = \beta_1 \cos\theta_i a_z$.

$k_i = \beta_1(\sin\theta_i a_x + \cos\theta_i a_z)$

$|k_i| = \beta_1$

$k_i = \beta_1(\sin\theta_i a_x + \cos\theta_i a_z)$

$r = x a_x + y a_y + z a_z$

$$E_{is} = E_{io}\left[\cos\theta_i a_x - \sin\theta_i a_z\right] e^{-jk_i r}$$

$$k r = \beta_1 (x\sin\theta_i + z\cos\theta_i)$$

$$E_{is} = E_{io}\left[\cos\theta_i a_x - \sin\theta_i a_z\right] \times e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)}$$

~~Eis~~ $$H_{is} = \frac{E_{io}}{\eta_1} e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)} a_y.$$

$$E_{rs} = E_{ro}(\cos\theta_r a_x + \sin\theta_r a_z) e^{-j\beta_1 (x\sin\theta_r - z\cos\theta_r)}$$

$$H_{rs} = -\frac{E_{ro}}{\eta_1} e^{-j\beta_1 (x\sin\theta_r - z\cos\theta_r)} \cdot a_y.$$

where $\beta_1 = \omega\sqrt{\mu_1\varepsilon_1}$

||y medium 2.

$$E_{ts} = E_{to}(\cos\theta_t a_x - \sin\theta_t a_z) e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)}$$

$$H_{ts} = \frac{E_{to}}{\eta_2} e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)} \cdot a_y$$

where $\beta_2 = \omega\sqrt{\mu_2\varepsilon_2}$.

Requiring that $\theta_r = \theta_i$ and that the tangential components of E and H be continuous at the boundaries $z=0$, we get -

$$E_{is} + E_{rs} = E_{ts}.$$

$$H_{is} + H_{rs} = H_{ts}.$$

$$(E_{io} + E_{ro})\cos\theta_i = E_{to}\cos\theta_t \quad \text{— (1)}$$

~~$\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{1}{\eta_2}E_{to}$~~

~~Expressing E_{ro} and E_{to} in terms of E_{io}.~~

$$\cancel{\Gamma = \frac{E_{ro}}{E_{io}}} = \cancel{\eta_2} \; \frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{1}{\eta_2}E_{to} \quad \text{— (2)}$$

from equ (1),

$$E_{to} = \frac{E_{io}\cos\theta_i + E_{ro}\cos\theta_i}{\cos\theta_t} \quad \text{— (3)}$$

Substituting equ (5) in equ (2).

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{ro}\cos\theta_i + E_{io}\cos\theta_i}{\eta_2\cos\theta_t}.$$

$$\eta_2 E_{io}\cos\theta_t - \eta_2 E_{ro}\cos\theta_t$$
$$= \eta_1 E_{ro}\cos\theta_i + \eta_1 E_{io}\cos\theta_i.$$

$$\eta_2 E_{io}\cos\theta_t - \eta_1 E_{io}\cos\theta_i$$
$$= \eta_1 E_{ro}\cos\theta_i + \eta_2 E_{ro}\cos\theta_t.$$

$$E_{io}[\eta_2\cos\theta_t - \eta_1\cos\theta_i] = E_{ro}[\eta_1\cos\theta_i + \eta_2\cos\theta_t]$$

$$\frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}.$$

$$\gamma_{11} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}.$$

$$\Rightarrow (E_{io} + E_{ro})\cos\theta_i = E_{to}\cos\theta_t \quad \text{--- (1)}$$

$$\left(\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1}\right) = \frac{E_{to}}{\eta_2} \quad \text{--- (2)}$$

from equ (1)

$$E_{ro}\cos\theta_i = E_{to}\cos\theta_t - E_{io}\cos\theta_i.$$

$$E_{ro} = \frac{E_{to}\cos\theta_t - E_{io}\cos\theta_i}{\cos\theta_i} \quad \text{--- (3)}$$

substitute equ (3) in equ (2).

$$\frac{E_{io}}{\eta_1} - \left(\frac{E_{to}\cos\theta_t - E_{io}\cos\theta_i}{\eta_1\cos\theta_i}\right) = \frac{E_{to}}{\eta_2}.$$

$$E_{io} - \frac{E_{to}\cos\theta_t + E_{io}\cos\theta_i}{\cos\theta_i} = \frac{E_{to}\eta_1}{\eta_2}$$

$$\eta_2 E_{io}\cos\theta_i$$

$$\cancel{\eta_2 E_{io}} - \eta_2 E_{to}\cos\theta_t + \eta_2 E_{io}\cos\theta_i = \eta_1 E_{to}\cos\theta_i$$

$$\eta_2 E_{io} + \eta_2 E_i\cos\theta_i = \eta_1 E_{to}\cos\theta_i + \eta_2 E_{to}\cos\theta_t$$

$$\cancel{E_{io}[\eta_2 - \eta_2}$$

$$E_{io}[\eta_2\cos\theta_i + \eta_2\cos\theta_i] = E_{to}[\eta_1\cos\theta_i + \eta_2\cos\theta_t]$$

$$\frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}$$

These equations are called Fresnel's equation for parallel polarization.

In this equation, It is possible that $\Gamma_{\parallel} = 0$; under this condition there is no reflection ($E_{ro} = 0$) and the incident angle at which this takes place is called Brewster angle $\theta_{B\parallel}$ (polarizing angle)

The brewster angle is obtained by setting $\theta_t = \theta_{B\parallel}$ when $\Gamma_{\parallel} = 0$.

$$\eta_2\cos\theta_t = \eta_1\cos\theta_{B\parallel}$$

$$\eta_2^2(1 - \sin^2\theta_t) = \eta_1^2(1 - \sin^2\theta_{B\parallel})$$

$$\sin^2\theta_{B\parallel} = \frac{1 - \frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}}{1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2}$$

when $\mu_1 = \mu_2 = \mu_0$,

$$\tan\theta_{B\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

Q): The skindepth of copper (Cu) at 3GHz in 2μm. Calculate the skin depth at 3GHz for another conductor whose conductivity is $\frac{1}{10}$ times that of Cu.

$$\delta = \frac{1}{\alpha}.$$

$$\delta_{cu} = 2\mu m.$$

$$\frac{1}{\delta_{cu}} = \alpha_{cu} = \sqrt{\frac{\sigma\mu\omega}{2}} = \sqrt{\frac{\sigma\mu 2\pi f}{2}} = \sqrt{\sigma\mu\pi f} = \frac{1}{2}.$$

$$\sigma_{cond} = \frac{1}{10} \sigma_{cu}.$$

$$\Rightarrow \alpha_{cond} = \sqrt{\frac{1}{10}\sigma_{c}\mu\pi f}.$$

$$= \sqrt{\frac{1}{10}} \times \frac{1}{2} = \frac{1}{2\sqrt{10}}$$

$$\delta_{cond} = \frac{1}{\alpha_{cond}} = 2\sqrt{10}$$

MODULE-4

POWER AND THE POYNTING VECTOR.

From Maxwell's equation.

$$\nabla \times E = \frac{-\partial B}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$B = \mu H$

$D = \varepsilon E$

$J = \sigma E.$

$$\nabla \times E = -\frac{\mu \partial H}{\partial t} \quad \text{——— (1)}$$

$$\therefore \nabla \times H = \sigma E + \frac{\varepsilon \partial E}{\partial t} \quad \text{——— (2)}.$$

Doting both sides of (2) with E.

$$E \cdot (\nabla \times H) = \sigma E^2 + E \cdot \frac{\varepsilon dE}{dt}.$$

$\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B)$

$\nabla \cdot (H \times E) = E \cdot (\nabla \times H) - H \cdot (\nabla \times E).$

Applying the property $E \cdot (\nabla \times H) = H \cdot (\nabla \times E) + \nabla \cdot (H \times E)$

$$H \cdot (\nabla \times E) + \nabla \cdot (H \times E) = \sigma E^2 + E \cdot \frac{\varepsilon \partial E}{\partial t}.$$

$\frac{\partial E^2}{\partial t} = 2E.$

$$= \sigma E^2 + \frac{\varepsilon}{2} \frac{\partial E^2}{\partial t} \quad \text{——— (3)}.$$

Doting both sides of (1) with H.

$$H \cdot (\nabla \times E) = -H \cdot \frac{\mu \partial H}{dt}.$$

$$= -\frac{\mu}{2} \frac{\partial H^2}{\partial t} \quad \text{——— (4)}.$$

Substitute (4) in (3).

$$-\frac{\mu}{2} \frac{\partial H^2}{dt} + \nabla \cdot (H \times E) = \sigma E^2 + \frac{\varepsilon}{2} \frac{\partial E^2}{dt}$$

$$-\frac{\mu}{2} \frac{\partial H^2}{\partial t} - \nabla \cdot (E \times H) = \sigma E^2 + \frac{\varepsilon}{2} \frac{\partial E^2}{\partial t}$$

Rearranging

$$\nabla \cdot (E \times H) = -\frac{\partial}{\partial t}\left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right) - \sigma E^2$$

Taking volume integral on both sides.

$$\int_V \nabla \cdot (E \times H)\, dv = -\frac{\partial}{\partial t}\int_V \left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right) dv - \int_V \sigma E^2 dv.$$

Applying divergence theorom on LHS.

$$\left[\oint_S (E \times H)\, ds = -\frac{\partial}{\partial t}\int_V \left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right) dv - \int_V \sigma E^2 dv\right]$$ - poynting's Theorem

Total power leaving the volume.

rate of decrease in energy stored in electric and magnetic fields

Ohmic power dissipation.

The quantity $E \times H$ is known as the poynting vector $\mathcal{P}$

$$\mathcal{P} = E \times H.$$

Poynting theorom states that the net power flowing out of a given volume v is equal to the time rate of decrease in the energy stored within v minus the ohmic losses

$$\boxed{\text{Time average power} = \frac{E_0^2}{2\eta} a_p = \frac{H_0^2 \, 2}{\eta} a_p}$$

Q) In free space. $E(x,t) = 50\cos(\omega t - \beta x)\, a_y$ V/m. Find the average power crossing area of radius 5m in the plane.

x = constant.

$$\text{Time avg power} = \frac{E_0^2}{2\eta} a_p \qquad E_0 = 50 \qquad \eta = 120\pi \text{ (free space)}.$$

$$= \frac{(50)^2}{2 \times 120\pi} a_p$$

$$= \frac{(50)^2}{2 \times 120\cancel{\pi}} \times \cancel{\pi}(5^2)\, a_p$$

$$= 260.42 \text{ watts.}$$

Q) In a non-magnetic medium $E = 4\sin(2\pi \times 10^7 t - 0.8x)\, a_z$ V/m,

Find

(i) ε_r, η

(ii) Time avg power carried by the wave.

(iii) Total power crossing $100\,cm^2$ of plane $2x + y = 5$.

Ans) lossless medium, $\beta = \omega\sqrt{\mu\varepsilon}$.

non-negative $\mu = \mu_0$

$$\beta = \omega\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}.$$

$\omega = 2\pi \times 10^7 \quad \beta = 0.8.$

$$0.8 = \frac{2\pi \times 10^7}{3 \times 10^8}\sqrt{\varepsilon_r}. \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\sqrt{\varepsilon_r} = \frac{0.8 \times 3 \times 10^8}{2\pi \times 10^7}$$

$$\varepsilon_r = \left(\frac{0.8 \times 3 \times 10^8}{2\pi \times 10^7}\right)^2 = 14.59$$

$= 15$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \times \frac{1}{\sqrt{\varepsilon_r}}$$

$$= \frac{\cancel{120\pi}\ 377}{\sqrt{14.59}} = 98.69\ \Omega$$

$$\approx 98.7\ \Omega$$

(ii) Time avg power $= \frac{E_0^2}{2\eta} = \frac{4^2}{2\times 98.7} = 0.081$

$= 81\times10^{-3}\ W/m^2$

$= 81\ mW/m^2$

(iii) Total power $= \left(81\times10^{-3}\ a_x \times 100\times10^{-4}\right)\left[\frac{2a_x + a_y}{\sqrt{5}}\right]$

~~$= 0.86\times10^{-3}\ a_x\left[2a_x + a_y\right]$~~

$= 0.72\ mW$

Q) A lossless dielectric medium has $\sigma = 0$, $\mu_r = 1$ & $\varepsilon_r = 4$. An EM wave has magnetic field components expressed as

$$H = -0.1\cos(\omega t - z)\,a_x + 0.5\sin(\omega t - z)\,a_y\ A/m.$$

a) Phase constant.
(c) Wave impedance
b) Angular velocity
(d) Electric field component.

Ans) a) $\beta = 1$ radian/meter.

b) $\beta = \omega\sqrt{\mu\varepsilon}$.

$$= \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r} \qquad = \frac{3\times10^8}{\sqrt{1\times4}} = 1.5\times10^8\ rad/sec$$

(c) $\eta = \sqrt{\dfrac{\mu}{\varepsilon}} = \sqrt{\dfrac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = \dfrac{377}{2} = \underline{\underline{188.4\,\Omega}}$

$\dfrac{E_0}{H_0} = \eta$

$E_{01} = H_{01}\eta = -0.1 \times 188.4$

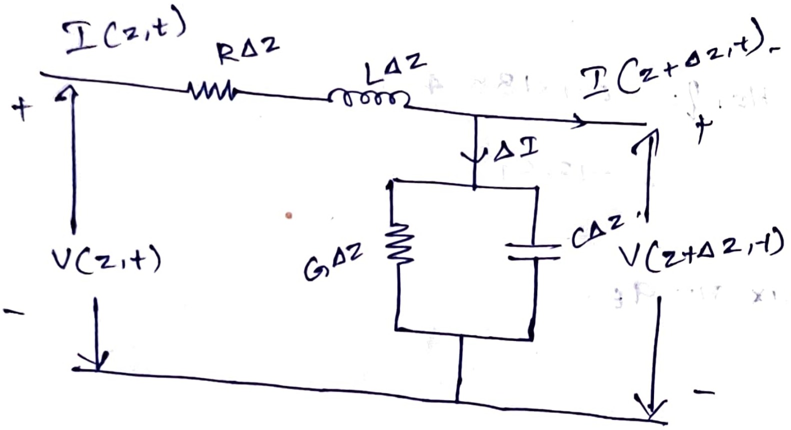
$= -18.84$

$a_H \times a_k = a_E$.

MODULE 4 [Continuation].

(Voltage current expression derivation)

L- Type equivalent Circuit.



Applying KVL to the outerloop of the circuit.

$$-V(z,t) + I(z,t)R\Delta z + L\Delta z \frac{\partial I(z,t)}{\partial t} + V(z+\Delta z,t) = 0.$$

$$-\left(V(z,t) - V(z+\Delta z,t)\right) + L\Delta z \frac{\partial I(z,t)}{\partial t} + I(z,t)R\Delta z = 0.$$

$$I(z,t)R\Delta z + L\Delta z \frac{\partial I(z,t)}{\partial t} = -\left(V(z+\Delta z,t) - V(z,t)\right)$$

$$-\frac{\left[V(z+\Delta z,t) - V(z,t)\right]}{\Delta z} = I(z,t)R + L\frac{\partial I(z,t)}{\partial t} \quad \text{——} (1)$$

Applying ~~limit~~ $\lim_{\Delta z \to 0}$ in equation (1), we get.

$$-\frac{\partial V(z,t)}{\partial z} = I(z,t)R + L\frac{\partial I(z,t)}{\partial t} \quad \text{——} (2)$$

Applying KCL.

$$I(z,t) = \Delta I + I(z+\Delta z,t).$$

$$= V(z+\Delta z,t)\,G\Delta z + C\Delta z\,\partial V...$$

$$-\left[I(z+\Delta z,t)-I(z,t)\right] = V(z+\Delta z)G\Delta z + C\Delta z\frac{\partial V(z+\Delta z,t)}{\partial t}$$

$$-\frac{\left[I(z+\Delta z,t)-I(z,t)\right]}{\Delta z} = V(z+\Delta z)G + C\frac{\partial V(z+\Delta z,t)}{\partial t} \quad (3)$$

$\Delta z \to 0 \Rightarrow$

$$-\frac{\partial I(z,t)}{\partial z} = V\cancel{(z+\Delta z)G + C\frac{\partial}{\partial t}V(z+\Delta z,t)}$$

$$-\frac{\partial I(z,t)}{\partial z} = V(z,t)G + C\frac{\partial}{\partial t}V(z,t) \quad (4)$$

$$-\frac{\partial V(z,t)}{\partial t} = R\,I(z,t) + L\frac{\partial I}{\partial t}(z,t)$$

$$-\frac{\partial I(z,t)}{\partial z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t}$$

If we assume harmonic time dependance.

$$V \to V_s \qquad \frac{\partial}{\partial t} = j\omega$$

$$-\frac{\partial}{\partial z}V_s = RI_s + Lj\omega I_s$$

$$-\frac{\partial}{\partial z}V_s = (R+j\omega L)I_s \quad (5)$$

$$-\frac{\partial}{\partial z}I_s = GV_s + Cj\omega V_s$$

$$-\frac{\partial}{\partial z}I_s = (G+j\omega C)V_s \quad (6)$$

differentiating (5) w.r.t. z,

$$-\frac{d^2}{dz^2}V_s = (R+j\omega L)\frac{d}{dz}I_s \quad (7)$$

substituting (6) in (7).

$$-\frac{d^2}{}V_s = (R+j\omega L)(G+j\omega C)V_s$$

$$-\left[I(z+\Delta z,t)-I(z,t)\right] = V(z+\Delta z)G\Delta z + C\Delta z\frac{\partial V(z+\Delta z,t)}{\partial t}$$

$$-\frac{\left[I(z+\Delta z,t)-I(z,t)\right]}{\Delta z} = V(z+\Delta z)G + C\frac{\partial V(z+\Delta z,t)}{\partial t} \quad \text{--- (3)}$$

$\Delta z \to 0 \Rightarrow$

$$-\frac{\partial I(z,t)}{\partial z} = V\cancel{(z+\Delta z)G + C\frac{\partial V(z+\Delta z,t)}{\partial t}}$$

$$-\frac{\partial I(z,t)}{\partial z} = V(z,t)G + C\frac{\partial}{\partial t}V(z,t) \quad \text{--- (4)}$$

$$-\frac{\partial V(z,t)}{\partial t} = RI(z,t) + L\frac{\partial I}{\partial t}(z,t).$$

$$-\frac{\partial I(z,t)}{\partial z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t}.$$

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differentiating (5) w.r.t. z,

$$-\frac{d^2}{dz^2}V_s = (R+j\omega L)\frac{d}{dz}I_s \quad \text{--- (7)}.$$

Substituting (6) in (7).

$$-\frac{d^2}{}V_s = (R+j\omega L)(G+j\omega C)V_s$$

let $\sqrt{(R+j\omega L)(G+j\omega C)} = \gamma$ (propogation constant)

$$\frac{d^2}{dz^2} V_s = \gamma^2 V_s.$$

$$\frac{d^2}{dz^2} V_s - \gamma^2 V_s = 0 \quad \text{——— (8)}$$

Similarly we will get

$$\frac{d^2 I_s}{dz^2} - \gamma^2 I_s = 0. \quad \text{——— (9)}$$

Solving (8) and (9)

$$V(z) = V_0^+ e^{\gamma z} + V_0^- e^{-\gamma z}$$

$$I(z) = \underbrace{I_0^+ e^{-\gamma z}}_{\text{+ve travelling wave}} + \underbrace{I_0^- e^{\gamma z}}_{\text{-ve travelling wave}}.$$

CHARACTERISTIC IMPEDANCE.

> It is defined as the ratio of the voltage to the current at any point of forward travelling wave.

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-}$$

$$-\frac{d}{dz} V_s = (R+j\omega L) I_s.$$

$$-\frac{d}{dz}\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right) = (R+j\omega L)\left(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}\right)$$

$$\gamma V_0^+ e^{-\gamma z} - \gamma V_0^- e^{\gamma z} = (R+j\omega L) I_0^+ e^{-\gamma z} + (R+j\omega L) I_0^- e^{\gamma z}.$$

Equating coefficients of $e^{-\gamma z}$,

$$\gamma V_0^+ = (R+j\omega L) I_0^+$$

$$\frac{V_0^+}{I_0^+} = \frac{R+j\omega L}{\gamma}$$

$$= \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

$$\boxed{Z_0 = \frac{V_0^+}{I_0^+} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}}$$

i) Lossless Line $[R = G = 0]$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC}$$

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}$$

$$\alpha = 0 \qquad \beta = \omega\sqrt{LC}.$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$= \sqrt{\frac{L}{C}}$$

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}}.$$

$$V_p = \frac{1}{\sqrt{LC}}$$

2) Distortionless line $\left[\frac{R}{L} = \frac{G}{C}\right]$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{R\left(1+j\omega\frac{L}{R}\right)}{G\left(1+j\omega\frac{C}{G}\right)}}$$

$$Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{R\left(1+j\omega\frac{L}{R}\right)G\left(1+j\omega\frac{C}{G}\right)}$$

$$= \sqrt{RG\left(1+j\omega\frac{C}{G}\right)^2}$$

$$\gamma = \sqrt{RG}\cdot\left(1+j\omega\frac{C}{G}\right)$$

$$\alpha+j\beta = \sqrt{RG} + j\omega\sqrt{\frac{R}{G}}\cdot C$$

$$\boxed{\alpha = \sqrt{RG}}$$

$$\beta = \omega\sqrt{\frac{R}{G}}\,C = \omega\sqrt{\frac{L}{C}}\cdot C$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

$$V = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

Q) An airline has a characteristic impedance of 70Ω and a phase constant of 3 rad/m at 100MHz. Calculate the inductance per meter & capacitance/m of the line.

Ans). airline → lossless line.

$Z_0 = 70\Omega$.

$\beta = 3 \text{ rad/m}$.

$Z = \sqrt{\frac{L}{C}}$ $\beta = \omega\sqrt{LC}$.

$Z_0\beta = \sqrt{\frac{L}{C}} \times \omega\sqrt{LC}$. $= \omega L$

$L = \frac{Z_0 \times \beta}{\omega}$

$= \frac{70 \times 3}{2\pi f} = \frac{70 \times 3}{2\pi \times 100 \times 10^6}$

$= \cancel{3.34}$ 334.2×10^{-9} H/m.

$= 334.2$ nH/m.

$\beta = \omega\sqrt{LC}$.

$\beta^2 = \omega^2 LC$.

$C = \frac{\beta^2}{\omega^2 L}$

$= \frac{3^2}{(2\pi \times 100 \times 10^6)^2 \times 334.2 \times 10^{-9}}$

$= 68.2 \times 10^{-12}$

$= 68.2$ pF/m

Q) A transmission line operating at 500MHz has.

$Z_0 = 80\Omega$; $\alpha = 0.04$ Np/m ; $\beta = 15$ rad/m. Find the line parameters.

Ans)

$$Z_0 \times \alpha = \sqrt{\frac{R}{G}} \times \sqrt{RG}$$

$$Z_0 \times \alpha = R$$

$$\cancel{R = \sqrt{Z_0 \times \alpha}}$$

$$R = Z_0 \times \alpha$$

$$= 80 \times 0.04 = 3.2\ \Omega/m$$

$$Z_0 = \sqrt{\frac{R}{G}}$$

$$Z_0^2 = \frac{R}{G}$$

$$G = \frac{R}{Z_0^2} = \frac{3.2}{(80)^2} = \cancel{0.5 \times 10^{-3}}$$

$$= .5 \times 10^{-4}\ S/m$$

$$\beta = \omega\sqrt{LC}$$

$$Z_0 \times \beta = \sqrt{\frac{L}{C}} \times \omega\sqrt{LC}$$

$$Z_0 \times \beta = L\omega$$

$$L = \frac{Z_0 \times \beta}{\omega} = \frac{80 \times 15}{2\pi \times 500 \times 10^{-6}} = 38.20\ H/m$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$C = \frac{L}{Z_0^2} = \frac{38.2 \times 10^{-9}}{(80)^2} = 5.97\ pF/m$$

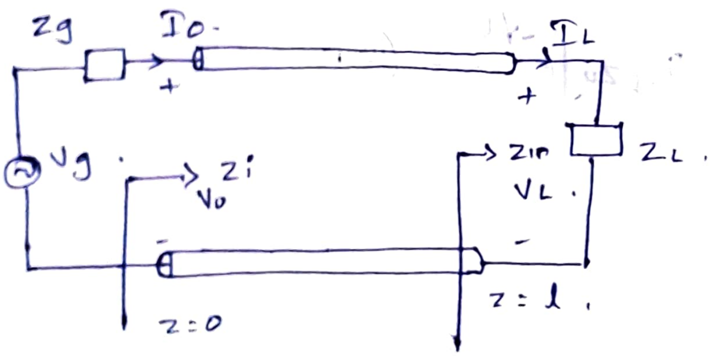
Lossless line.

A transmission line is said to be lossless, if the conductors of the line are perfect ($\sigma_c \simeq \infty$) and the dielectric medium seperating them is lossless ($\sigma = 0$) $[R = G = 0]$.

Distortionless line.

A distortionless line is one in which the attenuation constant α is frequency independent while the phase constant β is linearly dependant on frequency $\left[\frac{R}{L} = \frac{G}{C}\right]$

Input impedance of Transmission Line



$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad (1)$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-}$$

$$I_0^+ = \frac{V_0^+}{Z_0}$$

$$I_0^- = \frac{-V_0^-}{Z_0}$$

$$I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \quad (2)$$

At $z = 0$:

$$V_0 = V_0^+ + V_0^- \quad (3)$$

$$I_0 = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

$$I_0 Z_0 = V_0^+ - V_0^- \quad (4)$$

(3) + (4).

$$V_0^+ = \frac{1}{2}\left[V_0 + I_0 Z_0\right] \quad \text{--- (5)}.$$

(3) - (4)

$$V_0^- = \frac{1}{2}\left[V_0 - I_0 Z_0\right] \quad \text{--- (6)}.$$

At $z = l$:

$$V_L = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l} \quad \text{--- (7)}$$

$$I_L = \frac{V_0^+}{Z_0} e^{-\gamma l} - \frac{V_0^-}{Z_0} e^{\gamma l}$$

$$I_L Z_0 = V_0^+ e^{-\gamma l} - V_0^- e^{\gamma l} \quad \text{--- (8)}.$$

(7) + (8).

$$V_0^+ = \frac{1}{2}\left[V_L + I_L Z_0\right] e^{+\gamma l} \quad \text{--- (9)}.$$

(7) - (8).

$$V_0^- = \frac{1}{2}\left[V_L - I_L Z_0\right] e^{-\gamma l} \quad \text{--- (10)}$$

$$Z_{in} = \frac{V_s(z)}{I_s(z)}.$$

At the generator.

$$Z_{in} = \frac{V_0}{I_0} = \left[\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right] Z_0. \quad \text{--- (11)}.$$

substitute equ (9) in (11).

$$Z_{in} = \frac{V_0}{I_0} = \left[\frac{(V_L + I_L Z_0) e^{+\gamma l} + (V_L - I_L Z_0) e^{-\gamma l}}{(V_L + I_L Z_0) e^{+\gamma l} - (V_L - I_L Z_0) e^{-\gamma l}}\right] Z_0.$$

$$V_L = I_L Z_L.$$

$$Z_{in} = \left[\frac{(I_L Z_L + I_L Z_0)e^{+\gamma l} + (I_L Z_L - I_L Z_0)e^{-\gamma l}}{(I_L Z_L + I_L Z_0)e^{+\gamma l} - (I_L Z_L - I_L Z_0)e^{-\gamma l}}\right] Z_0$$

$$Z_{in} = \left[\frac{(Z_L + Z_0)e^{+\gamma l} + (Z_L - Z_0)e^{-\gamma l}}{(Z_L + Z_0)e^{+\gamma l} - (Z_L - Z_0)e^{-\gamma l}}\right] Z_0.$$

$$Z_{in} = \left[\frac{Z_L(e^{\gamma l} + e^{-\gamma l}) + Z_0(e^{\gamma l} - e^{-\gamma l})}{Z_L(e^{\gamma l} - e^{-\gamma l}) + Z_0(e^{\gamma l} + e^{-\gamma l})}\right] . Z_0$$

$$Z_{in} = \left[\frac{Z_L 2\cosh\gamma l + Z_0 2\sinh\gamma l}{Z_L 2\sinh\gamma l + Z_0 2\cosh\gamma l}\right] Z_0, \qquad \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \cosh\gamma l$$

$$\frac{e^{\gamma l} - e^{-\gamma l}}{2} = \sinh\gamma l.$$

$$Z_{in} = \left[\frac{Z_L\cosh\gamma l + Z_0\sinh\gamma l}{Z_L\sinh\gamma l + Z_0\cosh\gamma l}\right] Z_0,$$

dividing numerator and denominator by $\cosh\gamma l$.

$$Z_{in} = \left[\frac{Z_L + Z_0\tanh\gamma l}{Z_L\tanh\gamma l + Z_0}\right] Z_0,$$

where, l = length of transmission line.

For a lossless line $\alpha = 0$ $\therefore \gamma = \alpha + j\beta = j\beta$.

$$\tanh\gamma l = \tanh(j\beta)l.$$
$$= j\tan\beta l.$$

$$Z_{in} = \left[\frac{Z_L + jZ_0\tan\beta l}{jZ_L\tan\beta l + Z_0}\right] Z_0.$$

VOLTAGE REFLECTION COEFFICIENT

Voltage reflection coefficient at any point on the line is the ratio of the reflected voltage wave to that of the incident wave.

$$V_s(z) = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l}$$

$$\Gamma = \frac{V_0^- e^{-\gamma l}}{V_0^+ e^{\gamma l}}$$

$$\Gamma = \frac{V_0^-}{V_0^+} e^{2\gamma l}$$

$$\Gamma = \cancel{\left(\frac{V_L - I_L Z_0}{V_L + I_L Z_0}\right) e^{2\gamma l}}$$

$$\Gamma = \frac{(V_L - I_L Z_0) e^{-\gamma l}}{(V_L + I_L Z_0) e^{\gamma l}} \cdot e^{2\gamma l}$$

$$\Gamma = \frac{V_L - I_L Z_0}{V_L + I_L Z_0}$$

$$\Gamma = \frac{I_L Z_L - I_L Z_0}{I_L Z_L + I_L Z_0} \qquad V_L = I_L Z_L$$

$$\boxed{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

Z_L - Load impedance

Z_0 - Characteristic impedance.

Standing Wave Ratio.

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}}$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$Z_{in}/_{max} = \frac{V_{max}}{I_{min}} = \frac{V_{max}}{V_{min}/Z_0}$$

$$\boxed{Z_{in}/_{max} = SZ_0}$$

$$Z_{in}/_{min} = \frac{V_{min}}{I_{max}} = \frac{V_{min}}{V_{max}/Z_0} = \frac{Z_0}{S}$$

$$\boxed{Z_{in}/_{min} = \frac{Z_0}{S}}$$

Shorted line $(Z_L = 0)$.

$$Z_{in} = \left[\frac{Z_L + jZ_0\tan\beta l}{jZ_L\tan\beta l + Z_0}\right]$$

$$Z_{sc} = Z_{in}/_{Z_L=0} = \left(\frac{jZ_0\tan\beta l}{\cancel{Z_0}}\right)\cancel{Z_0}$$

$$Z_{sc} = \underline{jZ_0\tan\beta l}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_{sc} = \frac{-Z_0}{Z_0} = -1$$

$$S = \frac{1+|-1|}{1-|-1|} = \infty$$

$\cancel{\Gamma_{sc}}$

$\boxed{\Gamma_{sc} = -1}$ $\boxed{S_{sc} = \infty}$

Open Circuited Line ($Z_L = \infty$)

$$Z_{in} = \left[\frac{Z_L + jZ_0 \tan\beta l}{jZ_L \tan\beta l + Z_0}\right] Z_0.$$

$\cancel{Z_{oc} =}$

dividing ~~throughout~~ RHS by Z_L.

$$Z_{oc} = \left(\frac{1 + j\frac{Z_0}{Z_L}\tan\beta l}{\frac{Z_0}{Z_L} + j\tan\beta l}\right) Z_0.$$

$$= \left(\frac{1}{j\tan\beta l}\right) Z_0$$

$$Z_{oc} = \cancel{Z_0}\left(-j\cot\beta l\right) Z_0$$

$$\Gamma_{oc} = \frac{Z_L - Z_0}{Z_L + Z_0}.$$

$$= \frac{1 - \frac{Z_0}{Z_L}}{1 + \frac{Z_0}{Z_L}}$$

$$= \underline{\underline{1}}$$

$\cancel{Z_{oc} = \frac{1}{j\tan}}$

$S = \frac{1+|1|}{} = \infty$

Matched Line $(Z_L = Z_0)$

$$Z_{in} = \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] Z_0$$

$$\boxed{Z_{in} = Z_0}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = 1$$

Q) Find the reflection coefficient and VSWR [standing wave ratio] of a transmission line of characterist impedance 50Ω and load impedance j50Ω.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{j50 - 50}{j50 + 50} = \frac{j-1}{j+1} = \frac{(-1+j)}{(1+j)}$$

$$= \frac{1.414\angle 135}{1.414\angle 45}$$

$$= \underline{\underline{1\angle 90}}$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1}{1-1} = \infty$$

Q) A transmission line with the characteristics impedance of 300Ω. Is terminated in a purely reactive load. It is found by measurement that the minimum line voltage, upon it is 5μV and maximum 7.5μV. What is the value of load impedance.

$V_{min} = 5\mu V$

$V_{max} = 7.5\mu V$.

$$S = \frac{V_{max}}{V_{min}} = \frac{7.5\mu V}{5\mu V} = 1.5$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\boxed{\Gamma = \frac{V_{max}-V_{min}}{V_{max}+V_{min}}}$$

$$1.5 = \frac{1+\Gamma}{1-\Gamma}$$

$$1.5 - 1.5\Gamma = 1+\Gamma$$

$$0.5 = 2.5\Gamma.$$

$$\Gamma = \frac{0.5}{2.5} = 0.2.$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}.$$

$$0.2 = \frac{Z_L - 300}{Z_L + 300}.$$

$$0.2Z_L + 60 = Z_L - 300.$$

$$0.8Z_L = 360.$$

$$\underline{\underline{Z_L = 450\Omega}}.$$

Q) A load of 100 + j150 Ω is connected to a 75 Ω lossless line. Find reflection coefficient and standing wave ration.

$Z_L = 100 + j150\,\Omega$.

$Z_0 = 75\,\Omega$.

Normalized Load impedance $\bar{Z}_L = \dfrac{Z_L}{Z_0}$

$= \dfrac{100 + j150\,\Omega}{75}$

$= 1.33 + j2$

$|$reflection coefficient$| = \dfrac{|OP|}{|OQ|}$

$= \dfrac{5}{7.7} = 0.65$

$\Gamma = 0.65 \angle 40$

Standing wave ratio = 5

Zin at 0.4λ from the load.

$\boxed{\lambda = 720}$

Q) In a lossless transmission line with $Z_0 = 50\Omega$ is 30m long and operates at 2MHz. The line is terminated with a load $Z_L = 60 + j40\Omega$ if $\mu = 0.6c$ on the line find.

(i) Reflection coefficient
(ii) Standing wave ratio
(iii) Input impedance.

$Z_0 = 50 \quad l = 30 \quad F = 2MHz \quad Z_L = 60 + j40.$

$\mu = 0.6c.$

$$\bar{Z}_L = \frac{Z_L}{Z_0} = \frac{60 + j40}{50} = 1.2 + 0.8j.$$

$$|\Gamma| = \frac{|OP|}{|OQ|} = \frac{2.8}{8} = 0.35$$

$\Gamma = 0.35\angle 56.$

SWR = 2

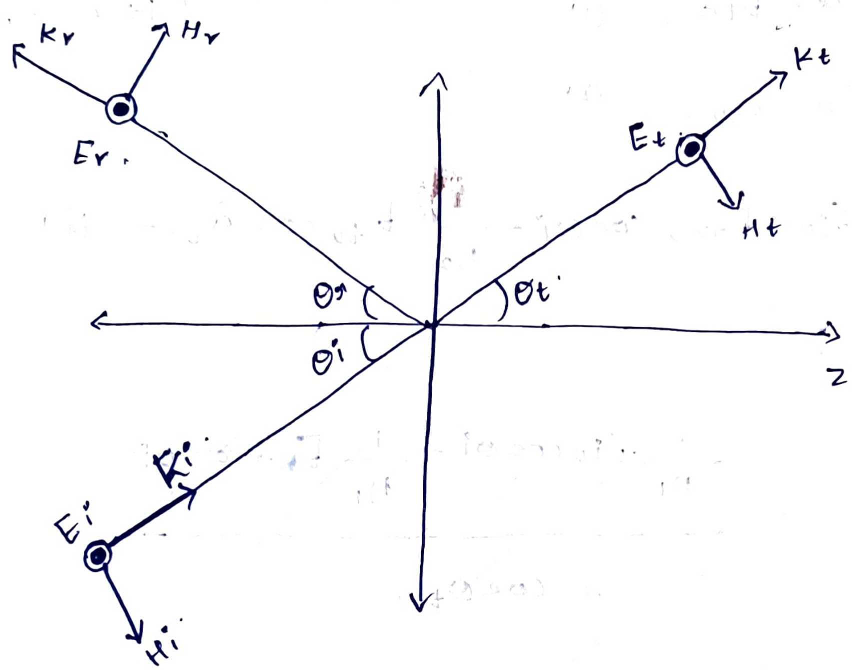
$$\lambda = \frac{\mu}{f} = \frac{0.6(3\times10^8)}{2\times10^6} = 90m.$$

$$l = 30m. = \frac{30}{90}\lambda = \frac{1}{3}\lambda.$$

$$= \frac{720^\circ}{3} = 240^\circ$$

PERPENDICULAR POLARISATION

Electric field is perpendicular to the plane of incidence.



In medium 1 :-

$$E_{is} = E_{i0}\, e^{-j\beta_i (x \sin\theta_i + z\cos\theta_i)}\, a_y.$$

$$H_{is} = \frac{E_{i0}}{\eta_1}\left(-\cos\theta_i\, a_x + \sin\theta_i\, a_z\right) e^{-j\beta_1 (x \sin\theta_i + z\cos\theta_i)}$$

~~In me~~

$$E_{rs} = E_{r0}\, e^{-j\beta_r (x\sin\theta_r + z\cos\theta_r)}\, a_y.$$

$$H_{rs} = \frac{E_{r0}}{\eta_1}\left(\cos\theta_r\, a_x + \sin\theta_r\, a_z\right) e^{-j\beta_1 (x\sin\theta_r + z\cos\theta_r)}$$

In medium 2.

$$E_{ts} = E_{t0}\, e^{-j\beta_t (x\sin\theta_t + z\cos\theta_t)}\, a_y.$$

$$H_{ts} = \frac{E_{t0}}{\eta_2}\left(-\cos\theta_t\, a_x + \sin\theta_t\, a_z\right) e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)}$$

Requiring that $\theta_r = \theta_i$.

tangential components of E & H be continuous at the boundaries z = 0, we get.

$$E_{is} + E_{rs} = E_{ts} \quad \text{--- (1)} \qquad H_{is} + H_{rs} = H_{ts} \quad \text{--- (3)}$$

$$E_{io} + E_{ro} = E_{to} \quad \text{--- (2)}$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro})\cos\theta_i = \frac{1}{\eta_2} E_{to}\cos\theta_t \quad \text{--- (4)}$$

$$\cancel{\frac{E_{to}}{\eta_2} = \frac{\frac{1}{\eta_1}E_{io}\cos\theta_i - \frac{1}{\eta_1}E_{ro}\cos\theta_i}{\eta_1 \cos\theta_t}}$$

$$\cancel{\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}}$$

$$\cancel{E_{ro} = r_\perp E_{io}}$$

(4).

$$\frac{E_{to}}{\eta_2} = \frac{E_{io}\cos\theta_i - E_{ro}\cos\theta_i}{\eta_1 \cos\theta_t} \quad \text{--- (5)}$$

(2) in (5).

$$\frac{E_{io}}{\eta_2} + \frac{E_{ro}}{\eta_2} = \frac{E_{io}\cos\theta_i - E_{ro}\cos\theta_i}{\eta_1\cos\theta_t}$$

$$\eta_1 E_{io}\cos\theta_t + \eta_1 E_{ro}\cos\theta_t = \eta_2 E_{io}\cos\theta_i - \eta_2 E_{ro}\cos\theta_i$$

E_{ro}.

$$\eta_1 E_{ro} \cos\theta_t + \eta_2 E_{ro} \cos\theta_i$$

$$= \eta_2 E_{io} \cos\theta_i - \eta_1 E_{io} \cos\theta_t.$$

$$\gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\cancel{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} \ \eta_2 \cos\theta_i + \eta_1 \cos\theta_t}.$$

Transmission coefficient $(\tau_\perp)$

$$\frac{1}{\eta_1}(E_{io} - E_{ro}) \cos\theta_i = \frac{1}{\eta_2} E_{to} \cos\theta_t.$$

$\theta_i = \theta_t$.

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}.$$

$$(E_{io} + E_{ro}) \cos\theta_i = E_{to} \cos\theta_t$$

$$E_{ro} = \frac{E_{to} \cos\theta_t - E_{io} \cos\theta_i}{\cos\theta_i}$$

$$\frac{E_{io}}{\eta_1} - \left(\frac{E_{to}\cos\theta_t - E_{io}\cos\theta_i}{\eta_1 \cos\theta_i}\right) = \frac{\eta_1 E_{to}}{\eta_2}.$$

$$\frac{E_{io}}{\eta_1} - \cancel{E_{to}\cos\theta_t}.$$

$$\eta_2 E_{io} \cos\theta_i - \cancel{\eta_1 E_{to} \cos\theta_t + \eta_2 E_{io} \cos}$$

$$\eta_2 E_{to} \cos\theta_t + \eta_2 E_{io} \cos\theta_i = \eta_1 E_{to} \cos\theta_i.$$

$$\eta_2 E_{io} \cos\theta_i + \eta_1\eta_2 E_{io} \cos\theta_i = \eta_1 E_{to} \cos\theta_i + \eta_1\eta_2 E_{to} \cos\theta_t$$

$$E_{io}\left[\eta_2 \cos\theta_i + \eta_2 \cos\theta_i\right]$$

$$= E_{to}\left[\eta_1 \cos\theta_i + \eta_2 \cos\theta_t\right].$$

$$\tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2n_2\cos\theta_i}{n_1\cos\theta_i + n_2 \cancel{\cos}\cos\theta_t}$$

Brewster angle.

$$r_\perp = 0 \qquad \theta_i = \theta_{B\perp}$$

$$0 = \frac{n_2\cos\theta_{B\perp} - n_1\cos\theta_t}{n_1\cos\theta_{B\perp} + n_2\cos\theta_t}$$

$$n_2\cos\theta_t = n_1\cos\theta_{B\perp}$$

$$n_2^2(1-\sin^2\theta_t) = n_1^2(1-\sin^2\theta_{B\perp})$$

$$n_1 = c\sqrt{\mu_1\varepsilon_1}$$

$$n_2 = c\sqrt{\mu_2\varepsilon_2}$$

$$\sin^2\theta_{B\perp} = \frac{1-\frac{\mu_1\varepsilon_2}{\mu_2\varepsilon_1}}{1-\left(\frac{\mu_1}{\mu_2}\right)^2}$$

Q) A 70Ω lossless line has $s = 1.6$ $\theta_r = 360°$

The line 0.6λ long obtain

Γ, Z_L, Z_{in}.

In a lossless transmission line,

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}.$$

Normalizing $\overline{Z_L} = \frac{Z_L}{Z_0}$.

dividing numerator and denominator by Z_0.

$$\Gamma_L = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1}$$

$$\Gamma_L = \frac{\overline{Z_L} - 1}{\overline{Z_L} + 1}$$

$$\Gamma_L(\overline{Z_L} + 1) = (\overline{Z_L} - 1)$$

$$\Gamma_L \overline{Z_L} + \Gamma_L = \overline{Z_L} - 1$$

$$1 + \Gamma_L = \overline{Z_L}(1 - \Gamma_L)$$

$$\overline{Z_L} = \frac{1 + \Gamma_L}{1 - \Gamma_L} \quad \text{—— (1)}$$

$$r + jx = \frac{1 + (\Gamma_r + j\Gamma_i)}{1 - (\Gamma_r + j\Gamma_i)}$$

$$= \frac{(1 + \Gamma_r + j\Gamma_i)}{1 - (\Gamma_r + j\Gamma_i)} \times \frac{(1 - \Gamma_r - j\Gamma_L)}{(1 - \Gamma_r - j\Gamma_L)}$$

Adding and subtracting with $\left(\frac{r}{r+1}\right)^2$.

$$\left(\frac{r-1}{r+1}\right) - \left(\frac{r}{r+1}\right)^2 + \left(\frac{r}{r+1}\right)^2 - \frac{2r\Gamma_r}{r+1} + \Gamma_r^2 + \Gamma_i^2 = 0.$$

$$\cancel{(r-1)\,(r+1}$$

$$\frac{(r-1)(r+1) - r^2}{(r+1)^2} + \Gamma_r^2 + \left(\frac{r}{r+1}\right)^2 - \frac{2r\Gamma_r}{r+1} + \Gamma_i^2 = 0$$

$$\frac{-1}{(r+1)^2} + \left(\Gamma_r - \frac{r}{r+1}\right)^2 + \Gamma_i^2 = 0.$$

$$\left(\Gamma_r - \frac{r}{r+1}\right)^2 + \Gamma_i^2 = \frac{1}{(r+1)^2}.$$